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Semiclassical Elementary Particle Models

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Abstract

Some simple models of elementary particles are discussed; they may be described as semiclassical, quark, shell models. Particles are assumed to be composed of spherical concentric charged shells. Three basic types of shell are allowed, quantum numbers are associated with each type such as to establish a quantum number correspondence between the shell types and the (p, n, λ) quarks. Particles are identified through the quantum numbers of their constituent shells (quarks).

The basic assumptions underlying the models considered are relationships between the electromagnetic energy associated with elementary particles (quark systems) and particle masses. The electromagnetic energy is represented classically; the models are semiclassical in that the shell radii are related to particle Compton wavelengths.

Particle mass and magnetic moment formulas are derived, possible values for quark masses are suggested, and possible connections of the models considered with particle symmetry schemes are discussed.

1. *Introduction*

Two models for elementary particles will be considered, together with a type of generalization applicable to both. These models have several properties in common. Elementary particle sub-structures are approximated by representing particles as sets of concentric spherical charged shells. Different types of shell are allowed, a shell type being defined by a set of quantum numbers. Basically three types, denoted by the symbols p, n, λ are considered; their associated quantum numbers are displayed in Table 1. These shell-type quantum number assignments correspond exactly to those associated with the (p, n, λ) quarks; for this reason a shell of type (p, n, λ) will be considered synonymous with a (p, n, λ) quark in the following. As usual, baryons will be built from three quarks, mesons from quark–antiquark pairs. Thus shell types $(\bar{p}, \bar{n}, \bar{\lambda})$, obtained from (p, n, λ) by changing the sign of the quantum numbers, will also be allowed. Finally, the muon will be built from leptonic quarks denoted as p^*, n^*, these being objects

TABLE 1

Shell types p, n and λ and their associated quantum numbers. The quantum numbers are identical with the charge, hypercharge, isospin, strangeness and baryon number quantum numbers originally associated with the p, n and λ quarks

	q	Y	I	I_z	S	B
p	2/3	1/3	1/2	1/2	0	1/3
n	−1/3	1/3	1/2	−1/2	0	1/3
λ	−1/3	−2/3	0	0	−1	1/3

with charge $2e/3$ and $-e/3$ respectively and lepton number 1/3. In general a particle is identified by its quantum numbers, which are the sums of the corresponding quantum numbers of its constituent quarks.

Other common aspects of the models considered are assumptions which relate particle electromagnetic energies to particle masses and shell radii to particle Compton wavelengths. These assumptions lead to particle mass formulas.

In Section 2 the relationship between the electrostatic energy associated with a particle and the sizes of its constituent shells is derived. Sections 3 and 4 discuss two elementary particle models which differ basically in the assumptions by which particle electromagnetic energies are related to particle masses. Section 5 discusses the possible connections of these models with particle symmetry schemes. In Section 6 the models considered in Sections 3 and 4 are generalized by including in the expression for the particle electromagnetic energy also contributions from magnetic dipole moments associated with the shells. Expressions are obtained for both particle masses and magnetic moments. Finally, the conclusions inferred by the results obtained are discussed in Section 7.

2. *The Basic Formalism*

The electrostatic energy, W, associated with a particle is expressed in terms of its electric field $\vec{E}$ as†

$$W = \int d^3r\, E^2/8\pi \tag{2.1}$$

Assuming particles to be composed of static charged shells having radii R_i and charges $q_i e$, the field $\vec{E}$ is the sum of partial fields $\vec{E}_i$ where

$$\vec{E}_i = q_i e\vec{r}/r^3 \qquad r > R_i \tag{2.2a}$$

$$\vec{E}_i = 0 \qquad r < R_i \tag{2.2b}$$

† Gaussian units, and the natural units obtained by setting $\hbar = c = 1$, are used throughout this paper.

Considering a system of three shells (the most general case which will be needed in the following), the result of evaluating (2.1) with the fields (2.2a, b) may be expressed as

$$2w/e^2 = Q_1/R_1 + Q_2/R_2 + Q_3/R_3 \tag{2.3}$$

where for

$$R_1 \leqslant R_2 \leqslant R_3 \tag{2.4}$$

the Q_i are

$$Q_1 = q_1^{\,2} \tag{2.5a}$$

$$Q_2 = q_2^{\,2} + 2q_2 q_1 \tag{2.5b}$$

$$Q_3 = q_3^{\,2} + 2q_3 q_2 + 2q_3 q_1 \tag{2.5c}$$

Expressing the shell radii in inverse mass units through

$$F_i = m_i R_i \tag{2.6}$$

equation (2.3) may be written

$$m_1/W = F_1/Q_1(2/e^2 - (Q_2/F_2 m_2/W + Q_3/F_3 m_3/W)) \tag{2.7}$$

In the following sections possible interpretations of relations (2.6) will be investigated. In general, with these interpretations equation (2.7) will become a particle mass formula.

3. *Model I*

In the type of model being considered there are two masses or energies which may be associated with a particle: its mass, M, and W (that part of M corresponding to the particle's electromagnetic energy). A possible interpretation of relations (2.6) would be that they represent quantum conditions relating the shell radii to the masses M and W. In particular the identifications assumed for the m_i in (2.6) are

$$|m_2| = |m_3| = W \tag{3.1a}$$

$$m_1/W = M/m \tag{3.1b}$$

where m is the electron mass. Of course it is not at all obvious why the electron mass should appear in the identifications (3.1a, b). However this would be very natural if $W = m$ for all particles. This would imply that the electron mass is completely of electromagnetic origin but that electromagnetism contributes relatively little to the mass of particles heavier than the electron.

Assuming the identifications (3.1a, b), equation (2.7) becomes

$$M/m = (F_1/Q_1)(2/e^2 - Q_2/F_2 - Q_3/F_3) \tag{3.2}$$

where, from (2.4),

$$|F_2| \leqslant |F_3|, \qquad F_1/M \leqslant |F_2|/m$$

With the interpretation of relations (2.6) as quantum conditions, the F_i might be expected to be integers. In order to test this idea, possible values for the F_i have been determined empirically for some elementary particles. The procedure used is as follows. The quark content (number and types of shells) of a particle is determined such that the particle quantum numbers are the sums of the corresponding quark quantum numbers. A specific type is assigned to each shell; the corresponding shell (quark) charges will determine the Q_i in (2.5). Values are determined for the F_i such that the mass M calculated for the particle from (3.2) will be in reasonable agreement with its measured mass. It is noteworthy that depending on the number of shells (three are used for baryons, two for mesons), equation (3.2) can give several expressions for the same particle mass, depending on the choice of the assignment of the shell types (charges) to the shells.

Table 2 lists the F_i values determined for various elementary particles; the shell types (quark composition), the mass M as calculated from (3.2), the measured mass M^{exp} with its experimental error δM^{exp}, and $|M - M^{\text{exp}}|$ are shown for each particle.

Certain features of Table 2 would seem to be significant. The calculated masses are in good agreement with the measured masses. Indeed, all the calculated baryon masses (except the Ξ^-) are equal within experimental error to the measured masses of these particles. These mass values are obtained using only simple rational numbers for the F_i. This fact would permit the interpretation of relations (2.6) as quantum conditions, if the quantum units may be chosen appropriately. Thus, considering for example only the baryons, R_2 could be quantized in units $1/(10m)$. Similarly, R_3 could be quantized in units $1/m$ if $F_1 = 1$ rather than $F = 8/9$ were used for the Λ (this would decrease the mass by about ·2 MeV).

However, for several reasons such conclusions as to the significance of relations (2.6) must be considered to be premature. First of all, even if they are constrained to have 'simple' values, the F_i are not at all uniquely determined by the particle masses. Various sets of simple rational numbers may be assigned to the F_i such as to reproduce the measured particle masses as well as the set shown in Table 2 (this is partically due to the large experimental errors associated with some of the particle masses). The set of F_i in Table 2 were chosen because of the similarity of the various F_2 values and F_3 values; this favors the quantum number interpretation. But even if the set of F_i in Table 2 is accepted, the spectra of the F_i are not simple enough to provide conclusive evidence for the interpretation of the F_i as quantum numbers. This last point, together with the somewhat inesthetic negative values of some of the F_i in Table 2 (implying negative values for the corresponding m_i), would suggest that perhaps the present model is incomplete; in Section 6 a model in which all the F_i may be positive will be considered.

A final difficulty of the present model is that the F_i values from Table 2 would imply anomalously large shell radii R_2 and R_3; these radii are, in order of magnitude, electron Compton wavelengths (if $W = m$).

TABLE 2

The F_i and the corresponding particle masses, M, from equation (3.2). For Σ^+, F_2 is irrelevant since Q_2 in (3.2) is zero for the quark combination λ, p, p. The measured particle masses M^{exp}, errors δM^{exp}, e^2, and m are from Particle Data Group (1972)

	Quarks (1, 2, 3)	F_1	F_2	F_3	M (MeV)	$\|M-M^{exp}\|$	δM^{exp}
P	$p\ p\ n$	3	4/5	−2	938.2603	·0011	·0052
N	$p\ n\ n$	3	−1/5	−8	939·5539	·0012	·0052
Λ	$\lambda\ n\ p$	8/9	1/5	8/9	1115·65	·06	·05
Σ^+	$\lambda\ p\ p$	17/18		4	1189·48	·06	·11
Σ^0	$\lambda\ n\ p$	17/18	−4/5	8	1192.49	·01	·11
Σ^-	$\lambda\ n\ n$	17/18	−1/5	8	1197·38	·04	·10
Ξ^0	$\lambda\ \lambda\ p$	19/18	1/10	2	1315·39	·69	·7
Ξ^-	$\lambda\ \lambda\ n$	19/18	1/5	2	1321·05	·25	·15
$\pi^\pm$	$n\ \bar{p}$	1/9	1		139·598	·022	·011
$K^\pm$	$\lambda\ \bar{p}$	7/18	−4/9		493·76	·08	·10
η	$p\ \bar{p}$	16/9	−1/12		549·31	·51	·6
μ	$p^*p^*n^*$	1/3	−1	−8/3	105·6617	·0023	·0004

4. *Model II*

The model considered in Section 3 was based essentially on the identifications (3.1a, b). These identifications seemed to imply the assumption that the electromagnetic energy associated with all particles is equal to the electron mass: $W = m$. However it is this assumption that leads to the association of anomalously large radii $R \simeq 1/m$ with particles having mass $M \gg m$.

The model considered in this section is based on the assumption that the electromagnetic energy associated with a particle is simply related to its mass. Precisely, it is assumed that

$$KM = W = \int d^3r\, E^2/8\pi \tag{4.1}$$

where $K \leqslant 1$ is a rational number, possibly different for different particles. Identifying the masses m_i in the relations (2.6) as

$$|m_2| = |m_3| = M = W/K \tag{4.2a}$$

$$m_1 = M^2/m = WM/Km \tag{4.2b}$$

the mass formula obtained in this model is

$$M/m = KF_1/Q_1(2/e^2 - Q_2/KF_2 - Q_3/KF_3) \tag{4.3}$$

Equation (4.3) is essentially equivalent to (3.2); the results of this model will be identical with those of the model of Section 3 if the KF_i assume the values associated previously with the F_i in (3.2). However, in the present model the larger shell radii will be approximately $R = 1/M$.

It would seem that the only possible interpretation of the mass m_1 defined by (4.2b) would consist in identifying it with the mass of a quark. But if m_1 is a quark mass then, for consistency, m_2 and m_3 should also be quark masses. With this interpretation of the m_i, relations (2.6) seem more natural since each m_i refers specifically to the object with radius R_i.

Two possible points of view suggested by these ideas will be considered. The first would correspond to the statement that various possible 'mass states' are allowed for fractionally charged particles; among these states are those corresponding to elementary particle masses (4.2a) and those corresponding to the masses (4.2b). The second point of view would be that various mass states and charge states (also fractionally charged states) are possible for 'particles'; however there is not a unique correspondence between mass states and charge states.

At this point it is convenient to introduce the symbols M_2 and Z; M_2 denotes the mass states defined by (4.2b)

$$M_2 = M^2/m \tag{4.4}$$

and Z represents the right-hand side of equation (4.3):

$$Z = (KF_1/Q_1)(2/e^2 - Q_2/KF_2 - Q_3/KF_3) \tag{4.5}$$

From (4.4) and (4.3), $M_2/M = Z$, or

$$M_2/m = Z^2 \tag{4.6}$$

It is of some interest to consider the possibility that (4.1) might also be applicable for particles with mass M_2; that is

$$M_2 = W = \int d^3r\, E^2/8\pi \tag{4.7}$$

Using the identifications (4.2a, b) and defining $M_3 = M^3/m^2$, (4.7) leads to $M_3/M_2 = Z$, or

$$M_3/m = Z^3 \tag{4.8}$$

where the Z in (4.8) is identical with that in (4.6) if the KF_i/Q_i for the particle of mass M_2 are the same as those for the particle of mass M in (4.1). By iteration of this idea (substituting M_3 for M_2 in (4.7), etc.) an infinite sequence of states with identical values for the KF_i/Q_i and masses

$$Mn/m = Z^n \tag{4.9}$$

could be predicted. Actually there would be many such sequences, each containing one 'elementary particle' state. Where necessary for clarity in the following the mass of a member of a specific sequence will be denoted as $M_n(x)$ where x is the customary symbol for the elementary particle state contained in the sequence and n denotes the nth member, or state, in the sequence.

The elementary particles would correspond to the members of the sequences (4.9) with $n = 1$. The states with $n > 1$ will correspond to very large masses, the smallest such mass being $M_2(\mu) = 21{\cdot}8$ GeV, where μ denotes the muon sequence. Since it may be quite difficult to experimentally prove or disprove the existence of such masses, the more immediate practical importance of the sequences (4.9) would be related to the possibility of their extension to states with $n < 1$. The state with $n = 0$ would correspond to the electron and the largest mass value associated with states with $n < 0$ would be $M_{-1}(\mu) = {\cdot}0025$ MeV.

With such an extension, every sequence of states would contain the electron and may be considered to be composed of pairs of states whose associated masses obey the relation $M_n M_{-n} = m^2$.

5. *Possible Connections with Particle Symmetry Schemes*

Noting that by far the largest contribution to the particle masses comes from the first term on the right-hand side of equation (4.3), the approximate mass formula consisting of only this term is considered; thus, denoting the KF_1 in (4.3) by F_1,

$$M/m = 2F_1/Q_1 e^2 \tag{5.1}$$

Using the F_1 from Table 2 in (5.1), the different members of an isospin multiplet will have identical masses. Denoting the mass of the multiplet by the customary symbol for the multiplet (N here means nucleon), the baryon masses calculated from (5.1) may be seen to exactly satisfy the following relation:

$$1/2(N + \Xi) = 1/4(\Sigma + 3\Lambda) \tag{5.2}$$

This is of course the SU_3 octet mass formula. If this formula is used for the pseudoscalar mesons, determining the massses from (5.1) with the F_1 of Table 2 one obtains

$$1/2(K + \bar{K}) \cong 1/4(\pi + 3\eta)$$

$$7/2(2m/e^2) \cong (7/2 - 1/4)\, 2m/e^2$$

In this case, the SU_3 mass formula is not satisfied exactly, but the squares of the meson masses from (5.1) do exactly obey the 'quadratic mass formula'

$$1/2(K^2 + \bar{K}^2) = 1/4(\pi^2 + 3\eta^2) \tag{5.3}$$

The expressions (5.2) and (5.3) would suggest a possible relationship between symmetry schemes such as SU_3 and the present models: perhaps the assumptions made in deriving the SU_3 mass formula are in some way related to the approximation of (4.3) by (5.1).

The model considered in Section 4 would suggest a possible explanation for the appearance of the masses squared in (5.3). Approximating (4.6) by

$$M_2/m = (2F_1/Q_1 e^2)^2 \tag{5.4}$$

and using the F_1 for the mesons from Table 2, the M_2 from (5.4) exactly satisfy the linear mass relationship

$$1/2(M_2(K) + M_2(\bar{K})) = 1/4(M_2(\pi) + 3M_2(\eta))$$

this expression being identical with (5.3). Thus, both the baryon masses and the masses M_2 of the more massive quarks associated with the pseudo-scalar mesons (but not the mesons themselves) exactly obey the SU_3 linear mass formula in the approximations (5.1) and (5.4); this fact might be related to the common fermion nature of the baryons and quarks.

6. *Model III*

In this section the previously considered models are generalized by associating not only charges, but also magnetic moments with the elementary particle shell (quark) substructures. Thus, the rest frame electromagnetic energy associated with a particle becomes

$$W = \int d^3r(E^2 + B^2)/8\pi \tag{6.1}$$

where $\vec{B}$ is the rest frame magnetic field of the particle.

The field $\vec{B}$ is the sum of fields $\vec{B}_i$ arising from the magnetic dipole moments $\vec{\mu}_i$ associated with the individual shells or quarks. The fields $\vec{B}_i$ are assumed to have the form which would result if the moments $\vec{\mu}_i$ arose from rotation of the charged shells; thus

$$\vec{B}_i = 3(\vec{\mu}_i \cdot \vec{r})/r^5 - \vec{\mu}_i/r^3 \qquad r > R_i \tag{6.2a}$$

$$\vec{B}_i = 2\vec{\mu}_i/R_i^3 \qquad r < R_i \tag{6.2b}$$

The result of evaluating (6.1) using (6.2a, b) and (2.2a, b) may be expressed as

$$2W/e^2 = (Q_1 + A_1)/R_1 + (Q_2 + A_2)/R_2 + (Q_3 + A_3)/R_3$$

where the Q_i are given by (2.5a, b, c) and the A_i are

$$A_1 = 2\mu_1{}^2/(R_1 e)^2 \tag{6.3a}$$

$$A_2 = 2(\mu_2{}^2 + 2\vec{\mu}_2 . \vec{\mu}_1)/(R_2 e)^2 \tag{6.3b}$$

$$A_3 = 2(\mu_3{}^2 + 2\vec{\mu}_3 . \vec{\mu}_2 + 2\vec{\mu}_3 . \vec{\mu}_1)/(R_3 e)^2 \tag{6.3c}$$

Thus, (4.3), or (3.2), becomes

$$M/m = (KF_1/(Q_1 + A_1))(2/e^2 - (Q_2 + A_2)/KF_2 - (Q_3 + A_3)/KF_3) \tag{6.4}$$

In order to avoid the difficulties associated with the large shell radii typical of the model considered in Section 3, the present model assumes identifications for the m_i in (2.6) similar to those assumed in Section 4. Thus, $W = KM$ and

$$m_1 = M^2/m \tag{6.5a}$$

$$m_2 = m_3 = M \tag{6.5b}$$

In the following, the possibility of determining both the masses and the magnetic moments of the baryons within the present model will be investigated. In particular the baryons P, N and Λ will be considered since both the masses and the moments are well measured for these particles.

The baryon magnetic moments, $\vec{U}$, are expressed in terms of the baryon spins, $\vec{S}$, and 'g' factors as

$$\vec{U} = ge/2M\vec{S} \tag{6.6}$$

The similar representation

$$\vec{\mu}_i = g_i q_i e/2m_i \vec{s}_i \tag{6.7}$$

might be expected to be meaningful for the quark magnetic moments, where the $\vec{s}_i$ would be quark angular momentum vectors. For definiteness, the $\vec{s}_i$ are defined to be vectors with the length $s_i = \sqrt{(3)}/2$ appropriate to an object with angular momentum components $\hbar/2$.

Defining $\vec{\mu} = \sum \vec{\mu}_i$, the condition $\vec{U} = \vec{\mu}$ relates the magnitudes U and μ by

$$U = g\sqrt{(3)}\, e/4M = \mu = \sqrt{[(\sum \vec{\mu}_i)^2]}. \tag{6.8}$$

With the limited objective of obtaining a qualitative idea of how the magnetic properties of the quarks might contribute to the particle masses and moments, the relationships (6.3a, b, c) and (6.8) are simplified by assuming the quark magnetic moments to be mutually orthogonal:

$$\vec{s}_i . \vec{s}_j = \sqrt{(3)}/2\, \delta_{ij} \tag{6.9}$$

From (6.8), using (6.7) and (6.9),

$$g = \sqrt{[\sum (g_i q_i M/m_i)^2]}$$

Using (6.5a, b), with $M \gg m$, g may be approximated as

$$g = \sqrt{[(g_2 q_2)^2 + (g_3 q_3)^2]} \tag{6.10}$$

From (6.7), (6.9) and (2.6) the A_i defined by (6.3a, b, c) are

$$A_i = 3(g_i q_i/F_i)^2/8 \tag{6.11}$$

Using (6.6) and (6.10), the magnitude (expressed in nuclear magnetons) of a specific component, U_z, of the magnetic moment of a baryon of mass M would be

$$|U_z| = M_p/2M\sqrt{[(g_2 q_2)^2 + (g_3 q_3)^2]} \tag{6.12}$$

where M_p is the proton mass.

The expressions (6.4), (6.11) and (6.12) determine both the mass and the magnetic moment of a particle in terms of the quantities K, g_i and F_i. As previously, the F_i are theoretically undetermined; this is also independently true for the g_i. Values of the K, g_i and KF_i such that the P, N and Λ masses and magnetic moments as calculated from (6.4) and (6.12) are in reasonable

TABLE 3

The K, KF_i, g_i and the magnetic moments $|U_z|$ in nuclear magnetons from equation (6.12). The measured moments U_z^{exp} and their errors δU_z^{exp} are from Particle Data Group (1972)

	Quarks (1,2,3)	K	KF_1	KF_2	KF_3	g_1	g_2	g_3	$\lvert U_z \rvert$	U_z^{exp}	δU^{exp}
P	$n\,p\,p$	$\frac{1}{3}$	1	1	1	$2\sqrt{2}$	6	$3\sqrt{3}$	2.65	2·79	$\sim 10^{-5}$
N	$n\,p\,n$	$\frac{1}{9}$	1	$\frac{1}{3}$	$\frac{1}{3}$	$6\sqrt{2}$	$\frac{9}{2}$	8	2.00	−1.91	$\sim 10^{-4}$
Λ	$\lambda\,p\,n$	$\frac{1}{18}$	1	$\frac{1}{9}$	$\frac{1}{9}$	$6\sqrt{3}$	2	$\frac{8}{3}$	.674	−.67	.06

agreement with their measured values are shown in Table 3. This table displays the assumed quark composition, the magnitude, $|U_z|$, of the magnetic moment as calculated from (6.12), the measured magnetic moment, U_z^{exp}, and the error δU_z^{exp} associated with U_z^{exp} for the baryons P, N and Λ. The P, N and Λ masses calculated from (6.4) with (6.11) are identical with the calculated masses displayed in Table 2 for these particles.

From only three applications it is somewhat difficult to judge the possible merits of the present model. However some observations of probably general significance may be inferred from the results displayed in Table 3. First of all, the results for the magnetic moments are considerably worse than those for the masses; this may be due to the very simple model used for the moments. A positive feature of this model would correspond to the observation that it would seem to be completely unnecessary to allow negative values for the F_i (and m_i) in order to be able to accurately predict particle masses.

7. *Conclusions*

Some rather general predictions are implied by the considerations of the preceding sections. It would seem that electromagnetism plays an important role in the determination of the elementary particle mass spectrum; this would be especially true for the model of Section 4. The model of Section 4 would imply the existence of very large mass states (quark masses) and may be generalized to predict states corresponding to masses less than that of the electron. There is also rather general evidence that these models maybe related to particle symmetry schemes such as SU_3.

On a more detailed level, these models yield formulas for the elementary particle masses and magnetic moments. However these results are weakened by the appearance of theoretically undetermined constants in the formulas. Empirical determination of these constants leads to the perhaps significant conclusion that the elementary particle masses may be calculated to great accuracy in terms of simple rational numbers (the F_i and A_i), the fine structure constant and the electron mass; the particle magnetic moments involve the same rational numbers.

Numerous modifications of the models discussed here may be considered in order to try to explain the constants in the mass and moment formulas. Such modifications might involve the use of quarks with charges differing from those of Table 1, various quark charge distributions and/or quark magnetic moment couplings, the explicit inclusion of non-electromagnetic interactions, the use of particle symmetry schemes, etc. But what would seem to be really needed would be a relation (a quantum mechanical eigenvalue equation ?) which would determine the radial structure of elementary particles and the 'energy levels' associated with the constituent quarks.

Reference

Particle Data Group (1972). *Physics Letters*, **39B**, 1.